

# 1 Fourier analysis

## 1.1 Prerequisites

1. complex numbers
2. linear algebra
3. continuous functions, from real to real
4. continuous functions, from complex to complex
5. polynomial functions, sine and cosine functions
6. integration
7. series
8. power series

## 1.2 Transforms

### 1.2.1 Cake Transform: a bed-time story

Suppose all simple cakes are made of *eggs*, some *flour* and some *milk*. The description of a cake can then be:

$$cake_1 = 3 * E + 10 * F + 4 * M$$

where flour and milk are measured in dl.

The standard way of making a cake is to mix all ingredients, then bake it; we imagine there is a *mathematical operator*, a function, called *MixBake* that takes three values (ie number of eggs, quantity of flour and milk) and maps them onto cakes. The formal definition of such a function is then:

$$MixBake : EGGS \times FLOUR \times MILK \rightarrow CAKES$$

$$(e, f, m) \rightarrow c$$

Suppose now we have way (another *operator*) to decompose a cake into its original ingredients; we can call this function *UnCake*, defined as:

$$UnCake : CAKES \rightarrow EGGS \times FLOUR \times MILK$$

$$c \rightarrow (e, f, m)$$

given a cake this function will return three numbers: the number of eggs needed to create that cake, and the flour and milk needed, expressed in dl (see figure 1).

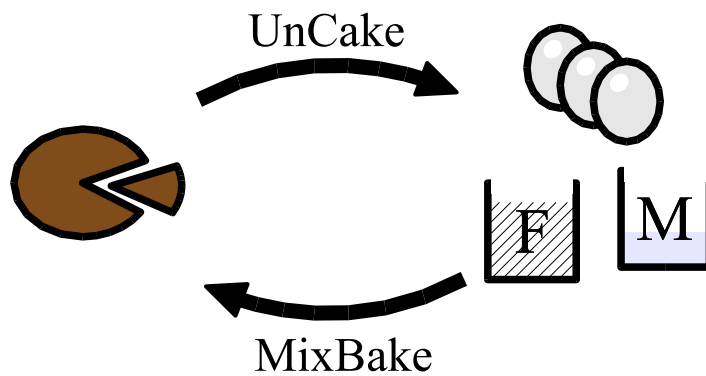


Figure 1: The Cake Transform

### 1.2.2 Cosine Transform

### 1.2.3 Fourier Transform

### 1.2.4 2D Cosin Transform

## 1.3 Discrete Transforms

### 1.3.1 Discrete Cosine Transform: DCT

### 1.3.2 Discrete Fourier Transform:DFT

## 1.4 Fast Fourier Transform: FFT

## 1.5 Applications

### 1.5.1 Filtering sound

### 1.5.2 Sound compression: mp3

### 1.5.3 Filtering images

### 1.5.4 Image compression: jpeg

### 1.5.5 Movie compression: mpeg

## 1.6 Further explorations

tutorial links

applets

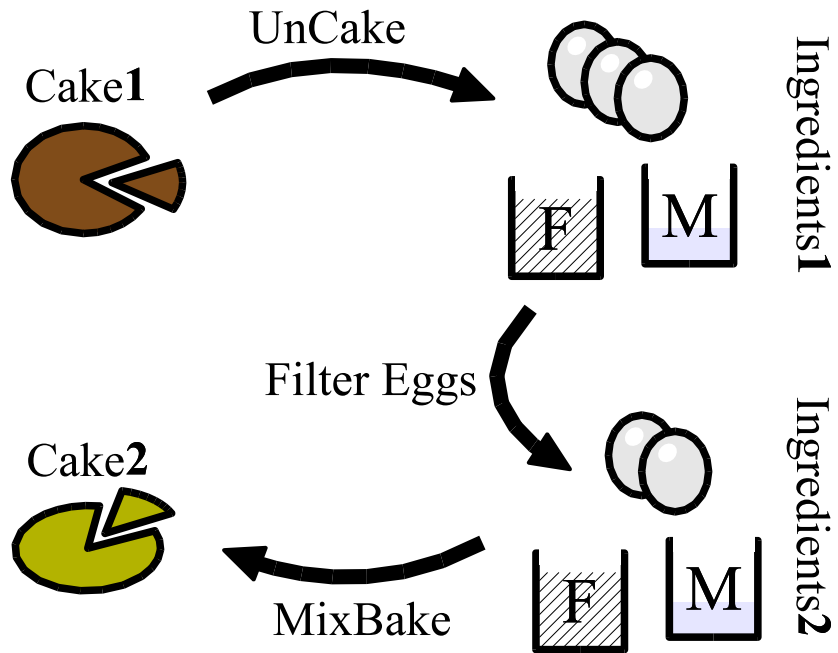


Figure 2: Filtering cakes

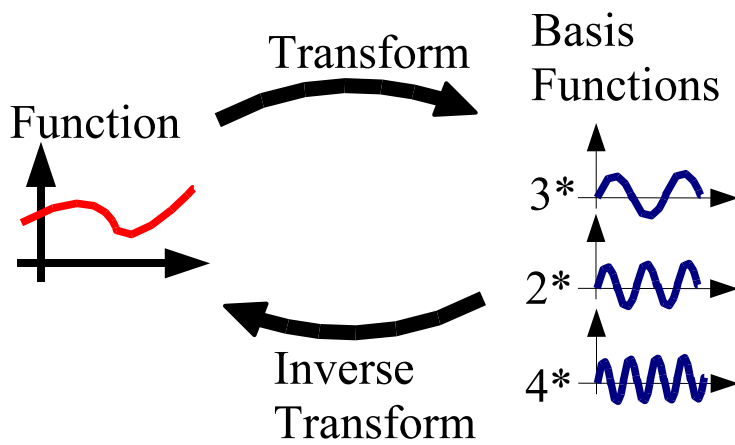


Figure 3: Functions

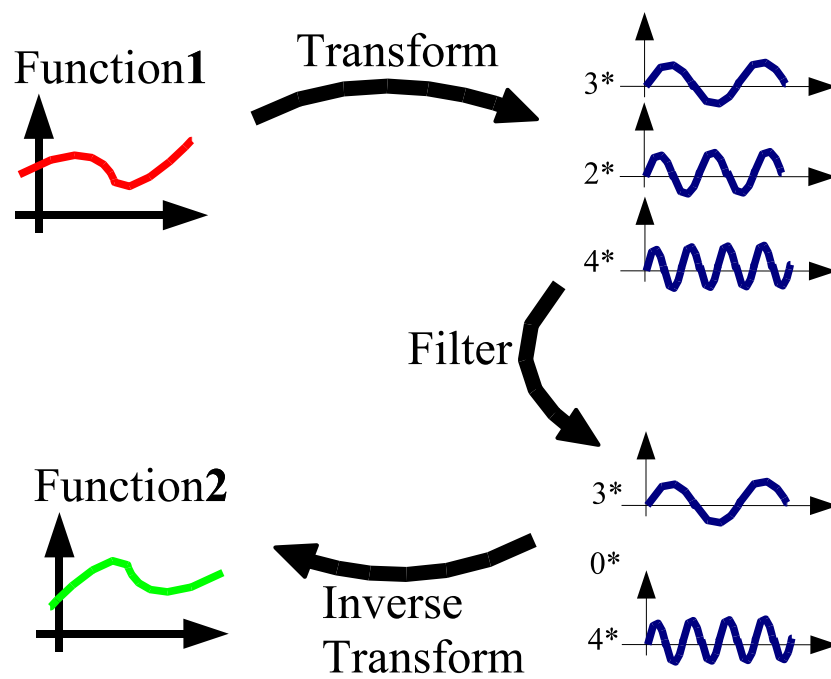


Figure 4: Filtering functions